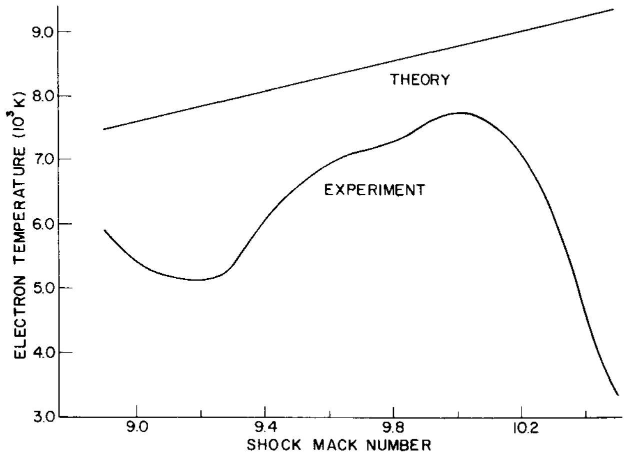

Fig. 3 Electron temperature of shock-heated argon initially at 4 torr and room temperature. The data for the experimental curve was obtained by Aro and Walsh[2] using a microwave-horn system whose *rf* was centered at 9.05 GHz; the curve incorporates the corrections of Ref. 4.

Conclusion

A basic limitation in microwave measurement of plasma temperature has been established: it has been shown that it is not possible to determine the electron temperature of shock-heated plasma by means of a microwave system operating at a frequency well below the plasma frequency because the plasma boundary layer exerts a prominent effect on the microwave measurements. The limitation applies, of course, to the determination of other plasma parameters as well, such as electron density and collision frequency. This limitation is responsible, in part, for the wide discrepancies between theory and experiment in the corrected[4] Aro-Walsh results[2] (Fig. 3) obtained by means of microwave horns for shock-heated argon at relatively high Mach numbers.

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Low Order Observer for a Linear Functional of the State Vector

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A PROCEDURE is described for the design of an observer of low order to provide a specified linear functional of the state vector of a linear system. The procedure yields information on the existence of an observer of given order, and on any constraints on the choice of observer poles.

There are many cases, e.g., in closed-loop pole assignment, where an observer is required to provide a specified linear functional of the state vector of a time-invariant linear system. A procedure for the design of such an observer, permitting the arbitrary choice of observer dynamics, was described in Ref. 2. However, as was shown by Fortmann and Williamson,[1] a reduction in observer order can be achieved by permitting the observer poles to be determined during the design process.

The method described by Fortmann and Williamson requires the reduction of the system to a number of single-output subsystems. In the present Note, a method is described, based on the procedure of Ref. 2, which is suitable for direct application to single-output or multioutput systems. The method permits the investigation of observers of increasing order, until an acceptable solution is found.

System Description

We consider a linear time-invariant system described by the equations

$$\begin{aligned} \dot{x} &= Ax+Bu \\ y &= Cx \end{aligned} \tag{1}$$

where x, u, and y are vectors of state, input, and output, of dimension n, r, and m, respectively, and A, B, and C are constant matrices. The pair (A, C) is observable.

Problem Statement

The problem is to design an observer described by the equation

$$\dot{z} = Dz+Ky+Gu \tag{2}$$

where z is the q-dimensional observer state vector, and D, K, and G are constant matrices, such that, for a specified n-vector h^T, (f^Ty+g^Tz) tends asymptotically to h^Tx. The matrices D, K, and G and the row vectors f^T and g^T are to be found such that D has acceptable, but not necessarily arbitrary, eigenvalues, and the dimension q of the observer, is to be as small as possible.

Procedure

We postulate the existence of an observer of order q, to provide a linear functional specified by the vector h^T. Let the characteristic polynomial of D be

$$s^q+\beta_{q-1}s^{q-1}+\ldots+\beta_1 s+\beta_0 \tag{3}$$

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Then, constraints on the coefficients β_i correspond to constraints on the choice of observer poles.

The following array is formed:

$$\begin{matrix} C \\ CA \\ \vdots \\ CA^q \\ h^T \\ h^T A \\ \vdots \\ h^T A^q \end{matrix} \tag{4}$$

We now perform a row reduction of the array [Eq. (4)]. The last $(q+1)$ rows are reduced in the process, but are not used in reducing other rows.

If the last $(q+1)$ rows are reduced to zero, there are no conditions on the β_i, and the observer poles can be chosen arbitrarily. Otherwise, each nonzero column in the last $(q+1)$ rows, say, $[\gamma_1 \ldots \gamma_{q+1}]^T$, provides a linear relationship among the β_i given by

$$\sum_{i=0}^{q-1} \beta_i \gamma_{i+1} + \gamma_{q+1} = 0 \tag{5}$$

Equation (5) provides a set of constraints which must be satisfied by the β_i. If this set is inconsistent, there is no solution for this value of q.

When a set of distinct observer poles has been chosen so as to satisfy the constraints, the design may be completed by following the procedure described in Ref. 2, substituting $(q+1)$ for p therein, wherever it occurs. The matrix U of column eigenvectors of D may be chosen arbitrarily, provided that the requirements of complex pairing are satisfied.

Proof

Postulating an observer of order q leads to the equations of Ref. 2, with $(q+1)$ replacing p. The necessary and sufficient condition for the existence of a solution is given by the condition for the consistency of Eqs. (8) of Ref. 2, viz., that the vector on the right-hand side lies in the space spanned by the rows of the coefficient matrix on the left-hand side. This coefficient matrix may be reduced by elementary row operations to the matrix

$$\begin{bmatrix} C \\ CA \\ \vdots \\ CA^q \end{bmatrix} \tag{6}$$

and the vector on the right-hand side may be written as

$$h^T A^q + \beta_{q-1} h^T A^{q-1} + \ldots + \beta_1 h^T A + \beta_0 h^T \tag{7}$$

where the β_i are the coefficients of the characteristic polynomial of D, as in Eq. (3). It follows that formation of the array [Eq. (4)] and the row reduction described yields the required constraints on the β_i.

Numerical Example

The procedure is illustrated by application to an example from Ref. 1. Here, C is a single row, c^T.

$$\begin{array}{lrrrrr} c^T & = 0 & 0 & 0 & 0 & 1 \\ c^T A & = 0 & 0 & 0 & 1 & -1.5 \\ c^T A^2 & = 0 & 0 & 1 & -1.5 & -1.25 \\ h^T & = 0.83 & -0.08 & -0.31 & 0.19 & 0.32 \\ h^T A & = -0.08 & -0.31 & 0.19 & 0.32 & -0.365 \\ h^T A^2 & = -0.31 & 0.19 & 0.32 & -0.365 & -0.432 \end{array} \tag{8}$$

Inspection reveals that no solution is possible if $q = 0$ or 1. With $q = 2$, row reduction in this case obviously leaves only the first two columns nonzero in the last three rows. This gives the condition

$$[-0.31 \quad 0.19] + \beta_1[-0.08 \quad -0.31] + \beta_0[0.83 \quad -0.08] = [0 \quad 0]$$

from which, $\beta_0 = 0.423$, $\beta_1 = 0.505$, and the observer poles are at $(-0.25 \pm j0.6)$, which agrees with the result in Ref. 1.

If we are not satisfied with these poles, we may consider a third-order observer, and this simply involves including the following extra rows in the appropriate places in the array [Eq. (8)]:

$$\begin{array}{lrrrrr} c^T A^3 & = 0 & 1 & -1.5 & -1.25 & 5.13 \\ h^T A^3 & = 0.19 & 0.32 & -0.365 & -0.432 & 0.906 \end{array}$$

Row reduction in this case leaves only the first column of the last four rows nonzero, and gives the condition

$$0.19 - 0.31\beta_2 - 0.08\beta_1 + 0.83\beta_0 = 0 \tag{9}$$

We may then specify observer poles at, say, -1 and -2, and an unknown λ. Inserting these in Eq. (9) gives $\lambda = -0.81$.

Conclusion

The method described, which is suitable for use with a computer, enables a linear functional observer of low order to be designed in a routine manner. The existence of a design of any given order is established, and the constraints imposed on the choice of observer poles are revealed.

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